Response to Appendix A of Shared Spectrum's Reply Comments

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Overview

In Appendix A of its Reply Comments, Shared Spectrum attempts to analyze the effect of additional interference on coverage and capacity of a CDMA CMRS system to show that the analyses "by QUALCOMM, Verizon Wireless, and allied parties in this proceeding [are] fundamentally faulty and premised on inappropriate application of a theoretical model" ([1], p. 1). Shared Spectrum further states that its Appendix A "provide[s] an accurate analysis of what is actually involved" (*id.*).

In fact, as shown here, Shared Spectrum's coverage analysis is technically flawed, suffers from obvious errors and omissions, is internally inconsistent, and demonstrates a fundamental lack of understanding of basic link budget analysis, and of CDMA system design.

Shared Spectrum's claim that an interference temperature 15 dB above the noise floor will not affect CDMA capacity is equally groundless. The correct capacity vs. interference temperature model is developed here, and it is shown that even relatively low levels of added interference have a significant effect on the reverse link capacity of a CDMA system. What Shared Spectrum does not address, and what is shown here, is that there is a tradeoff between coverage and capacity reduction when external interference is introduced. With added interference 15 dB above the noise floor, the coverage area of the CDMA reverse link would need to be dramatically reduced, to about 2-3% of its baseline value, for the capacity to be nearly "unchanged."

Shared Spectrum's Coverage Analysis

Shared Spectrum first summarizes the well-known mobile radio propagation model consisting of exponential median path loss increase with distance plus lognormal shadow fading ([1], p. 4):

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$$
\begin{aligned}
L_p(d) &= \overline{L}_p(d_0) + \varepsilon_{dB} \\
&= \overline{L}_p(d_0) + 10\gamma \log\left(\frac{d}{d_0}\right) + \varepsilon_{dB}, d \geq d_0
\end{aligned}
\qquad \text{([1], p. 4)}
$$

where d_0 is some reference distance and γ is the path loss exponent. There seems to be a typographical error on the first line of this equation, and the term $\overline{L}_p(d_0)$ should in fact be $\overline{L}_p(d)$.

The term ε_{dB} is a zero-mean Gaussian random variable that represents the effects of the shadow fading, and its probability density function (pdf) is

$$
f_{\varepsilon_{dB}}(x) = \frac{1}{\sigma_\varepsilon \sqrt{2\pi}} \exp\left(-\frac{x^2}{2\sigma_\varepsilon^2}\right) \qquad \text{([1], p. 4)}
$$

Shared Spectrum then gives an expression for "outage probability" which it defines as "the probability P_{out} that the received signal strength at location $d = r(> d_0)$ is below the threshold P_n". The parameter P_n is later defined as "background noise power" ([1], p. 13). The expression given is:

$$
P_{out} = \Pr(-10\gamma \log r + \zeta < P_n) = 1 - Q\left(\frac{P_n + 10\gamma \log r}{\sigma}\right) \qquad \text{([1], p. 5)}
$$

where $Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx$.

The outage expression is clearly incorrect, since it does not account for the transmitted power level, nor the required level of the received signal with respect to noise. Further, it does not even account for path loss properly. The correct expression would be

$$
P_{out} = \Pr\left\{P_{TX} - L_p(d_0) - 10\gamma \log\left(\frac{r}{d_0}\right) - \varepsilon_{dB} < K + P_n\right\}
$$

where P_{TX} is the transmitted power in dBm and K is the dB difference between the required received signal power and the noise floor P_n. Shared Spectrum's errors suggest a fundamental lack of understanding of basic link budget analysis.

Shared Spectrum then proceeds to compute the coverage reduction as a function of the interference temperature, which it defines as $I_T = P_n \Delta$ ([1], p. 6). That is, if I_{ext} is the added external interference, then Shared Spectrum's equations suggest that $\Delta = 1 + I_{ext}/P_n$, so $I_T = P_n + I_{ext}$. Since the noise is increased by a factor of Δ, so must the received desired signal power. Thus, the ratio of received signal power levels with and without interference is $S_w/S_{w/o} = \Delta$. For a given transmit power, received power varies with distance as $S \propto d^{-\gamma}$, so the ratio of maximum ranges with and without the added interference is $r_w/r_{w/o} = \Delta^{-1/\gamma}$ and the reduction in range is $1 - \Delta^{-1/\gamma}$, a result that Shared Spectrum obtains by a somewhat circuitous route involving the outage probability, although the end result is that the coverage reduction is independent of the outage probability.

Following the derivation, Shared Spectrum notes that "This expression is generally valid for <u>any</u> wireless cellular system based on any kind of radio access technology" ([1], p. 6, emphasis in original). However, despite this fact, Shared Spectrum then states: "QUALCOMM failed to specify the outage probability for which it claimed that the cell radius is reduced by about 20%. Its analysis is faulty ([1], p. 6)." It is unclear why Shared Spectrum would make such a statement when, by its own derivation, the coverage reduction is independent of the outage probability.

Shared Spectrum next attempts to compute the increase in outage probability due to the added interference, expressed as:

$$c_{out}(r,\Delta) = P_{out}(r, P_n\Delta) - P_{out}(r, P_n)$$
$$= Q\left(\frac{P_n(dB) + \Delta(dB) + 10\gamma \log r}{\sigma}\right) - Q\left(\frac{P_n(dB) + 10\gamma \log r}{\sigma}\right) \qquad \text{[1], p. 7}$$

This expression suffers from the same deficiencies as the basic outage formula discussed above. Shared Spectrum does not state its assumptions regarding transmit power or the intercept in the median path loss formula (i.e., $\overline{L}(d) = \overline{L}_p(d_0) + 10\gamma \log(d/d_0)$, where $\overline{L}_p(d_0) - 10\gamma \log d_0$ is the intercept). In any case, it is unclear why Shared Spectrum even attempted this calculation, since it had already derived the expression for the coverage reduction and found it to be independent of the outage probability. Thus, the curves presented by Shared Spectrum in Figs. 2-5 do not show what parameters were used, other than the path loss exponent, and do not provide any relevant information.

Despite the lack of supporting information on its own calculations, Shared Spectrum criticizes Qualcomm, asserting that

> QUALCOMM should clearly state the sizes of its cell coverage areas for urban, suburban, rural, indoor, etc., along with the outage probabilities as accurately as possible in light of the details for CDMA technologies. Our analysis reveals only upper bounds of system outage probabilities for any mobile technology in order to illustrate the fundamental mistake in QUALCOMM['s] analysis ([1], p. 11).

Given that Shared Spectrum has confirmed the applicability of an expression for coverage reduction that does not depend on the factors it lists, and that same expression was used by Qualcomm, the criticism of Qualcomm seems to have no logical or factual basis. Further, it is not clear what Shared Spectrum views as the "fundamental mistake."

Summary

Shared Spectrum's coverage analysis serves only to confirm the relationship already provided by Qualcomm and others. Shared Spectrum's attempts to relate coverage changes to interference temperature and outage probability are flawed and seem to suggest a fundamental lack of understanding of basic mobile radio link analysis. There seems to be no rational basis for Shared Spectrum's criticism of the Qualcomm analysis.

Shared Spectrum's Capacity Analysis

In its analysis of capacity ([1], pp. 12-16), Shared Spectrum addresses the capacity impact of additional external interference, claiming that even with "increasing the noise by 15 dB above the noise level, the CDMA capacity is unchanged" ([1], p. 16, emphasis in original). It is shown here that Shared Spectrum's results are incorrect unless the coverage area is greatly reduced.

Basic CDMA Reverse Link Relationships

The notation used here is summarized below and is consistent with that used in [1] and [2] (which is referenced as the source of the capacity formula in [1]).

E_b	Energy of received signal per information bit
I_0	One-sided interference plus noise power spectral density
P_n	Background noise power
S	Signal power per mobile received at cell site receiver
G_p	Processing gain
η_f	Frequency reuse efficiency
c_d	Capacity degradation factor due to imperfect power control
Q	Number of cell sectors
s_f	Source activity factor

It is assumed that there are N_{MS} mobiles per cell or sector which are intermittently transmitting with activity factor s_f. The total average received power from both in-cell and other-cell mobiles, plus noise, is

$$P_{tot} = \underbrace{s_f N_{MS} S}_{\text{in-cell}} + \underbrace{k_{oc} s_f N_{MS} S}_{\text{other-cell}} + P_n \qquad (1)$$

where k_{oc} is an other-cell load factor, which depends on geometry and propagation characteristics, and relative cell loading. The in-cell power is $P_{in} = s_f N_{MS} S$, the other cell power is $P_{out} = s_f k_{oc} N_{MS} S$, and P_n is thermal noise due to the receiver itself. The factor κ_f defined in [2] is equal to $\kappa_f = 1 + k_{oc}$, and (1) becomes:

$$P_{tot} = s_f N_{MS} S \kappa_f + P_n \ . \tag{2}$$

For the receiver associated with a given mobile, the power from other in-cell mobiles $s_f (N_{MS} - 1) S = P_{in} - s_f S$, and the total reverse link power from other mobiles, plus noise, is therefore

$$I = s_f S (N_{MS} \kappa_f - 1) + P_n = P_{tot} - s_f S \ . \tag{3}$$

With

$$E_b / I_0 = G_p \cdot S / I \ , \tag{4}$$

the number of mobiles per cell or sector can be expressed as

$$N_{MS} = \frac{\eta_f}{s_f} \left(\frac{G_p}{E_b / I_0} + s_f - \frac{P_n}{S} \right) \tag{5}$$

where $\eta_f = 1/\kappa_f$.

The degradation due to imperfect power control effectively increases the required E_b / N_0, but for purposes of evaluating the relative effect of external interference, the exact value of E_b / N_0 is not important, as will be seen shortly. Similarly, the number of sectors per cell can be ignored, since N_{MS} can be specified on a per-sector basis, and the relative capacity degradation again will be independent of the number of sectors. With $c_d = 1$ and $Q = 1$, eq. 6.4.15 of [2], which is the equation used by Shared Spectrum in [1], becomes:

$$N_{MS} = 1 + \frac{\eta_f}{s_f} \left(\frac{G_p}{E_b / I_0} - \frac{P_n}{S} \right) = \frac{\eta_f}{s_f} \left(\frac{G_p}{E_b / I_0} + s_f \kappa_f - \frac{P_n}{S} \right)$$

which is slightly different from (5).

Based on (5), the "pole capacity" is obtained for $P_n/S \to \infty$, giving:

$$N_{pole} = \frac{\eta_f}{s_f}\left(\frac{G_p}{E_b/I_0} + s_f\right), \tag{6}$$

and (5) can be written as

$$N_{MS} = N_{pole} - \frac{\eta_f}{s_f}\frac{P_n}{S} \ , \tag{7}$$

which can be rearranged to give

$$N_{pole}\frac{s_f}{\eta_f}\frac{S}{P_n} = \frac{1}{1 - N_{MS}/N_{pole}} \ . \tag{8}$$

Substituting for N_{pole} on the left hand side using (6) gives:

$$\frac{S}{P_n}\left(\frac{G_p}{E_b/I_0} + s_f\right) = \frac{1}{1 - N_{MS}/N_{pole}} \ . \tag{9}$$

Combining (3) and (4) gives

$$S\left(\frac{G_p}{E_b/I_0} + s_f\right) = P_{tot} \ . \tag{10}$$

Combining (9) and (10) gives the expression for the load curve shown in Figure 1:

$$\frac{P_{tot}}{P_n} = \frac{1}{1 - N_{MS}/N_{pole}} \ . \tag{11}$$

CDMA systems are engineered for a maximum value of P_{tot}/P_n (sometimes referred to as the "noise rise") of about 6 dB or less, to limit the required dynamic range of the front end amplifier at the base station (which must be highly linear), to limit the maximum required mobile transmit power, and to ensure system stability. Typically, the reference level P_n is constant, and is determined by the receiver hardware. For a 4 dB noise figure, for example, $P_n = -109\,\text{dBm}$. Letting $\lambda = N_{MS}/N_{pole}$ be the "load factor" and $\phi = P_{tot}/P_n$ represent the noise rise gives the relationship

$$\lambda = \frac{P_{tot} - P_n}{P_{tot}} = 1 - \frac{1}{\phi} \quad . \tag{12}$$

Note that the total signal power received at the base station from all CDMA mobiles is $P_s = P_{in} + P_{out}$, so $P_{tot} = P_s + P_n$. Hence, $\lambda = 1 - P_s/P_{tot}$.

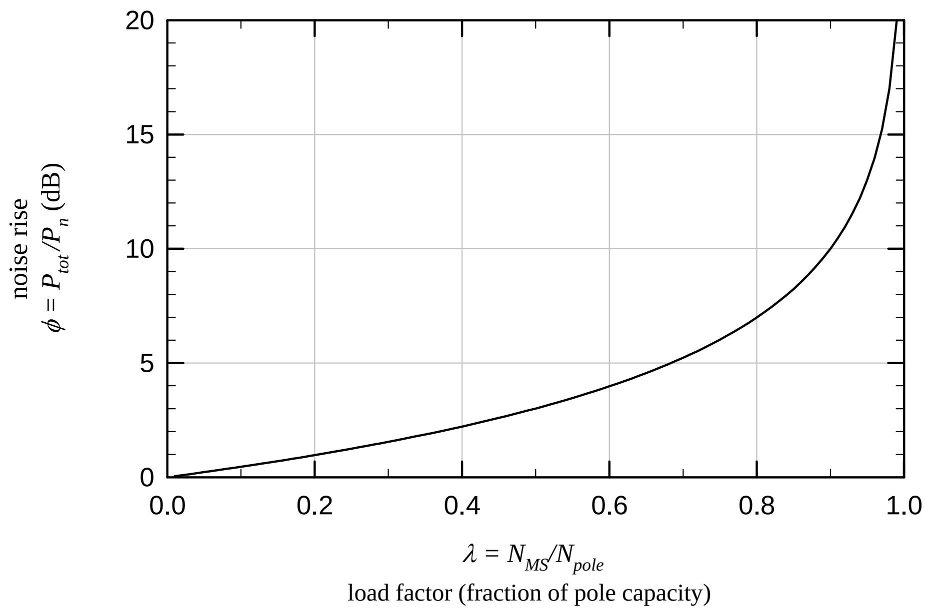

Figure 1: *CDMA uplink load curve.*

Effect of Added External Interference

Suppose that an external interference power $\Delta \cdot P_n$ is added at the CDMA base station receiver. Note that this definition of Δ (the same as in Shared Spectrum's capacity analysis) is different than the definition of Δ in Shared Spectrum's coverage analysis discussed above. Replacing P_n with $(1+\Delta)P_n$ but keeping P_{tot} the same gives

$$\lambda = \frac{P_{tot} - P_n(1+\Delta)}{P_{tot}} \quad . \tag{13}$$

Relative to the baseline load without interference, denoted λ_0, the load is:

$$\begin{aligned} \frac{\lambda}{\lambda_0} &= \frac{P_{tot} - P_n(1+\Delta)}{P_{tot} - P_n} \\ &= 1 - \frac{\Delta}{\phi - 1} \end{aligned} \quad . \tag{14}$$

Because $\lambda = N_{MS}/N_{pole}$, this is also the relative capacity N_{MS}; that is, if N_{MS0} is the number of mobiles per cell or sector that can be supported with no external interference (corresponding to a load of λ_0) then $N_{MS}/N_{MS0} = \lambda/\lambda_0$.

Figure 2 shows the CDMA reverse link capacity relative to the baseline vs. $10\log\Delta$ dB for $\lambda_0 = 0.75$ (corresponding to a noise rise of $\phi = 6\,\text{dB}$) and $\lambda_0 = 0.55$, corresponding to $\phi = 3.5\,\text{dB}$. It is clear from this figure that Shared Spectrum's claim that with "increasing the noise by 15 dB <u>above</u> the noise level, the CDMA capacity is unchanged" ([1], p. 16, emphasis in original) is not accurate if the noise rise P_{tot}/P_n is held constant. In fact, even if the added interference is well below the existing noise floor, the reverse link capacity of the CDMA system can still be reduced significantly, as demonstrated by Figure 2.

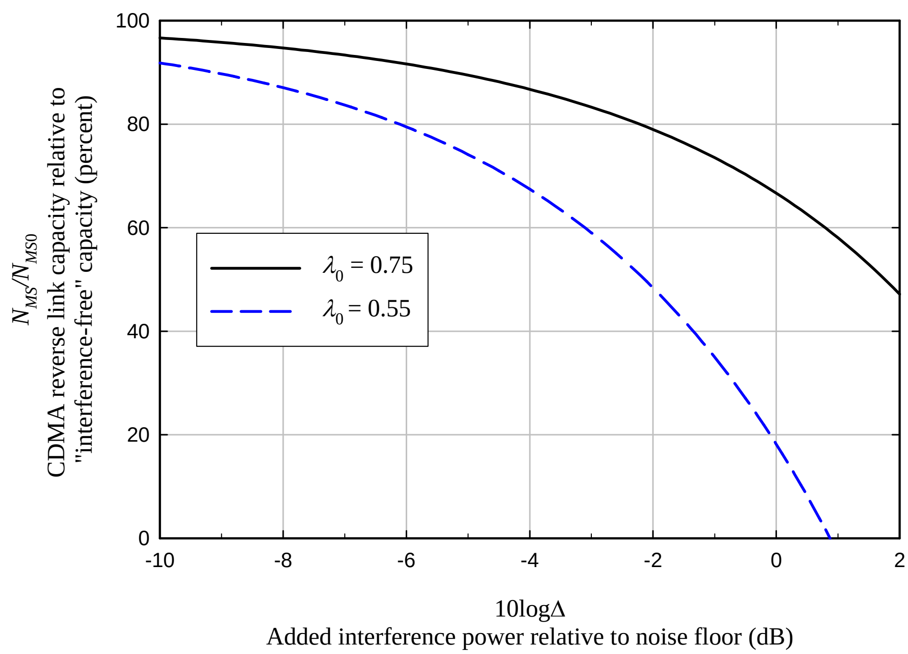

Figure 2: *CDMA capacity reduction vs. added interference relative to noise floor.*

The Capacity/Coverage Tradeoff

Shared Spectrum provides two three-dimensional plots attempting to show the capacity as a function of both the interference temperature and the received signal power (presumably corresponding to *S* in the equations) and on the basis of these results claims that interference 15 dB above the noise will not affect capacity. However, these results are highly misleading, because what Shared Spectrum does not explain is that coverage must be severely reduced.

To understand this, the relationship between capacity, coverage, and added interference can be shown by allowing the received signal power *S* from each mobile to increase to some new value $x \cdot S$, where $x > 1$, to compensate for the added external interference. Assuming that the upper limit on the maximum mobile transmit power does not change, this will reduce the coverage area by a factor of $A/A_0 = x^{-2/\gamma}$, where A_0 is the original coverage area and γ is the path loss exponent. If *S* changes by a factor of *x*, then so must P_{tot} according to (10). Thus, the load becomes:

$$\lambda = \frac{xP_{tot} - P_n(1+\Delta)}{xP_{tot}} \quad . \tag{15}$$

The ratio of the capacity with the added interference to that without it is:

$$\frac{N_{MS}}{N_{MS0}} = \frac{\lambda}{\lambda_0} = \frac{\phi - \left(\frac{1+\Delta}{x}\right)}{\phi - 1} \quad . \tag{16}$$

Hence,

$$\frac{N_{MS}}{N_{MS0}} = \frac{\phi - (1+\Delta)(A/A_0)^{\gamma/2}}{\phi - 1} \quad . \tag{17}$$

Note that as $A/A_0 \to 0$, $\lambda/\lambda_0 \to \dfrac{\phi}{\phi - 1} = N_{pole}/N_{MS0}$.

However, the relationship of (17) is unrealistic because it allows the total power at the base station receiver to increase without bound as the coverage area shrinks; Figure 3 shows the associated noise rise at the CDMA base station receiver as a function of the relative coverage area. To operate within the linear range of the base station receiver and to maintain system stability, the noise rise must be held at or below some threshold value, typically 6 dB or less.

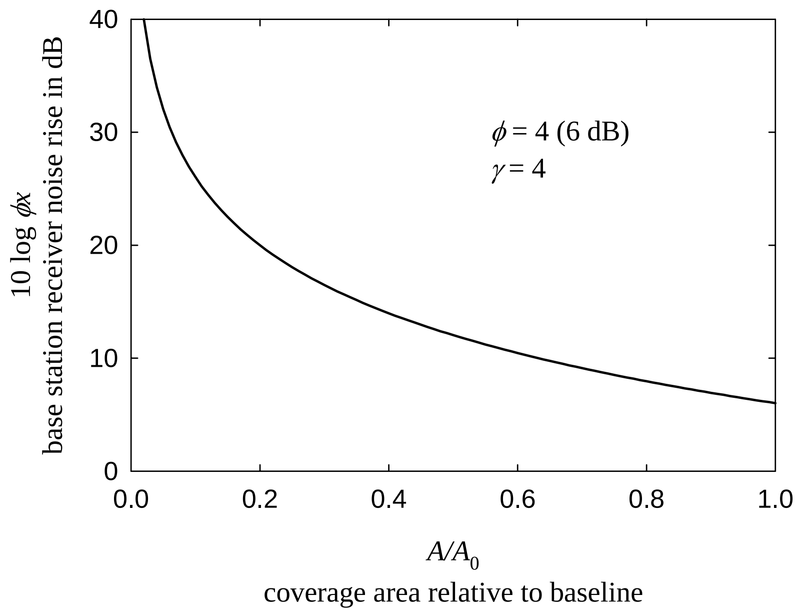

Figure 3: *Noise rise at the base station (P_{tot}/P_n) in dB vs. the fractional coverage.*

To incorporate this constraint into the coverage/capacity tradeoff analysis, the received signal power and interference will be reduced by a factor α, so the signal power received by the base station from each mobile is xS/α, and from (10) the total front-end power is xP_{tot}/α.[1] Thus, to maintain P_{tot} constant, $\alpha = x = \left(A/A_0\right)^{-\gamma/2}$ and the added interference is $\Delta \cdot P_n/\alpha$. The load is therefore

$$\lambda = \frac{P_{tot} - P_n\left[1 + \Delta\left(A/A_0\right)^{\gamma/2}\right]}{P_{tot}} \tag{18}$$

and the capacity relative to baseline is

$$\begin{aligned} \frac{N_{MS}}{N_{MS0}} &= \frac{\lambda}{\lambda_0} = \frac{P_{tot} - P_n\left[1 + \Delta\left(A/A_0\right)^{\gamma/2}\right]}{P_{tot} - P_n} = \frac{\phi - \left[1 + \Delta\left(A/A_0\right)^{\gamma/2}\right]}{\phi - 1} \\ &= 1 - \frac{\Delta\left(A/A_0\right)^{\gamma/2}}{\phi - 1} \end{aligned}. \tag{19}$$

[1] Physically, α represents an attenuator at the front end of the base station low noise amplifier. Actual implementation of such a scheme is not technically feasible because the attenuator would need to be automatically adjusted in real time according to the received external interference level, and this level could not be accurately measured in the presence of the signals from the CDMA handsets. Further, it would require a complete redesign of all base station hardware and control software, as well as the power control and admission control algorithms. Finally, it is obviously impractical to design a wireless network for which the coverage of each sector varies unpredictably in response to external interference, resulting in randomly varying coverage gaps.

Figure 4 shows N_{MS}/N_{MS0} vs. A/A_0 for different values of Δ according to (19), and Figure 5 shows the same relationship for $\Delta = 15\,\text{dB}$ with a log scale for the relative area. As can be seen, the coverage area must be reduced to about 2-3% of its baseline value for capacity to be relatively "unchanged" in this case.

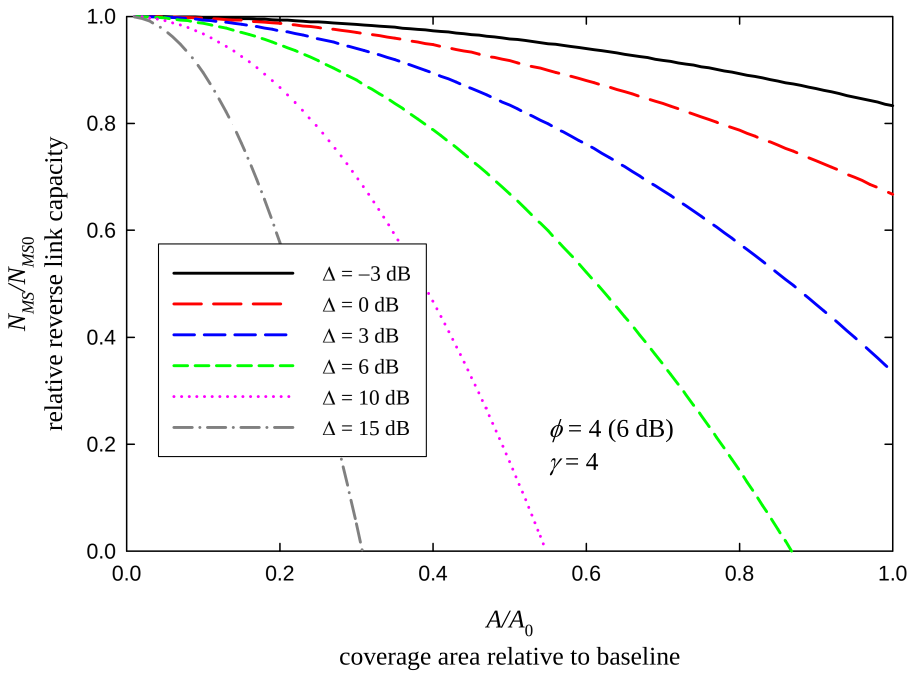

Figure 4: *Relative CDMA reverse link capacity vs. relative coverage for different levels of added interference (relative to noise floor), with attenuation at the base station receiver front end to maintain constant noise rise.*

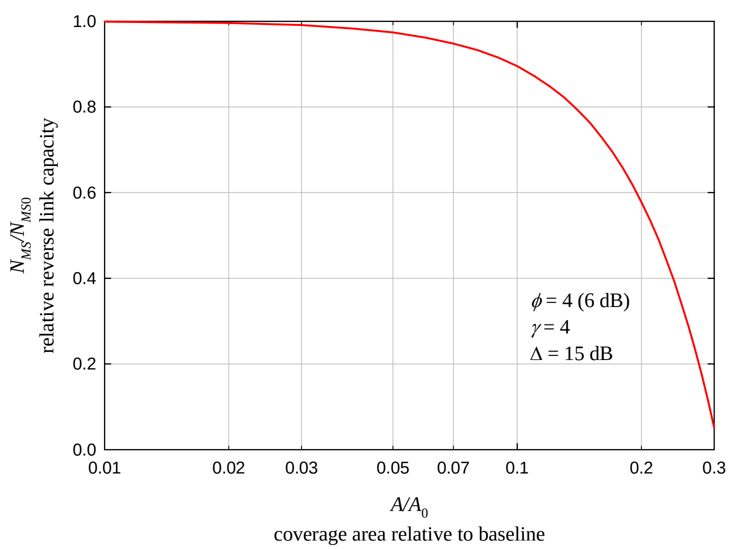

Figure 5: *Relative capacity vs. relative coverage for* $\Delta = 15\,dB$.

Figure 6 shows the same relationships as Figure 4, but on a 3D plot, for $\phi = 6\,\text{dB}$ and $\gamma = 4$. This curve looks quite different from those shown by Shared Spectrum in Figures 6 and 7 of [1], but the difference seems to be primarily a matter of scaling. For example, Figure 7 shows the same example as Figure 6, except that that upper limit of the relative coverage scale is 0.1 instead of 1.0, and a logarithmic scaling is used so that distance along the axis is linearly related to received power in dBm. As can be seen by comparing Figure 7 with Figure 8, (reproduced from Figure 6 of [1]), the two surfaces are very similar, suggesting that the same phenomenon is being represented.

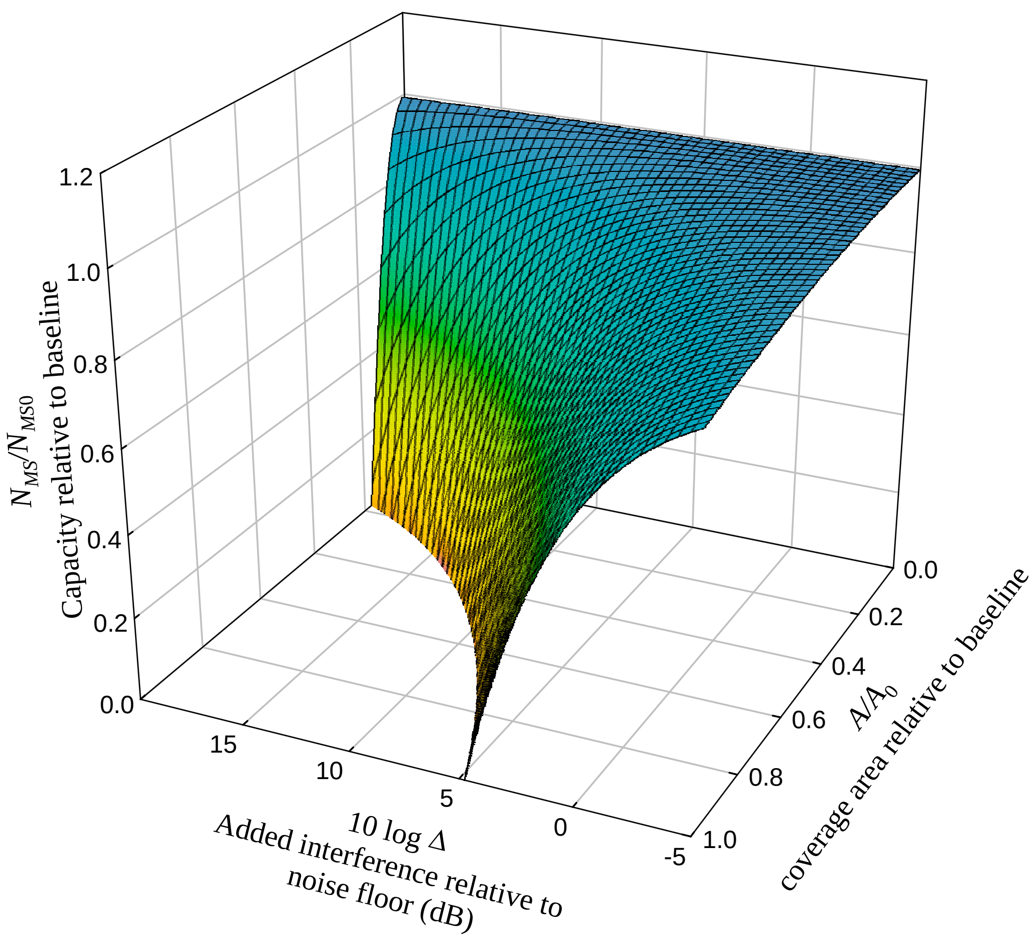

Figure 6: *Capacity vs. relative coverage and added interference.*

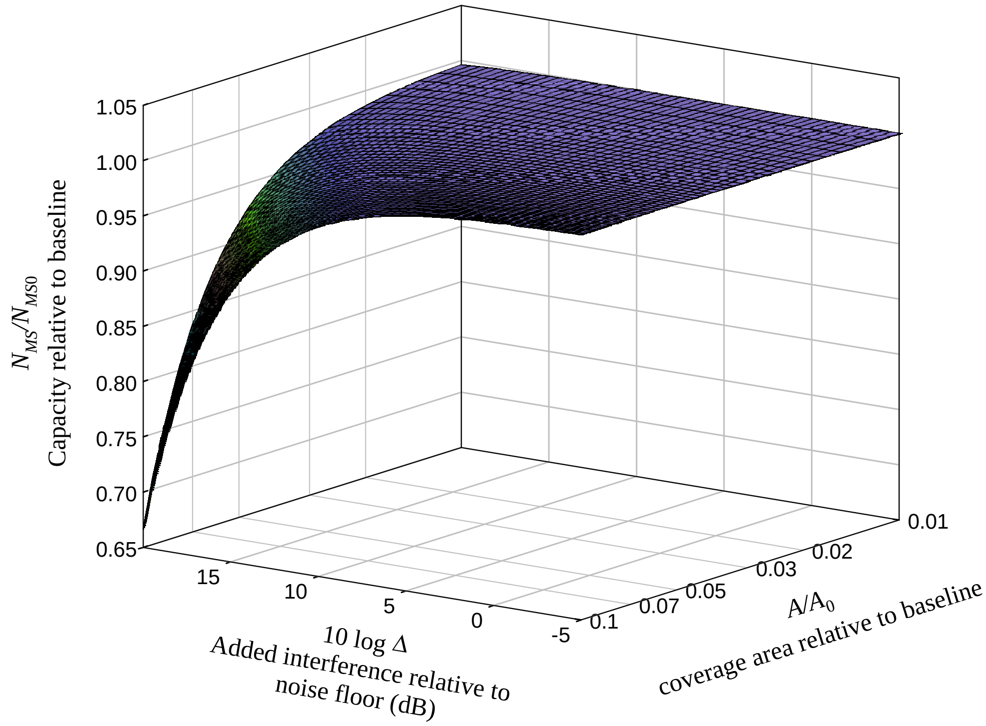

Figure 7: *Capacity vs. relative coverage and added interference, different scaling (compare with Figures 6 and 7 of [1]).*

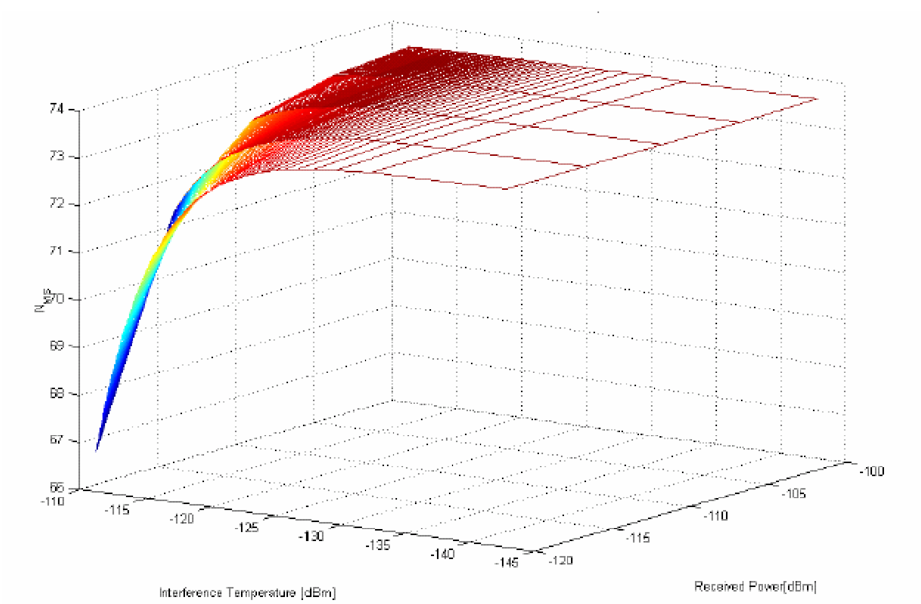

Figure 8: *Figure 6 from [1].*

Summary
The capacity results provided by Shared Spectrum are demonstrably incorrect if cell coverage is maintained constant. It is unclear what parameters were used in generating the numerical results presented by Shared Spectrum, including P_n, s_f, η_f, and the allowable noise rise at the base station receiver. However, the range used for the "interference temperature" axis in Figures 6 and 7 of [1], coupled with Shared Spectrum's reference to an interference temperature 15 dB above the noise floor, suggests that Shared Spectrum used the incorrect noise floor. With a bandwidth of 1.25 MHz and a 4-dB noise figure, the noise floor (P_n) would be about –109 dBm. The upper end of the "interference temperature" axis is –110 dBm, which is 1 dB below the noise floor rather than 15 dB above it. Moreover, Shared Spectrum states:

> In Figure 6, the system capacity remains 73 while the Interference Temperature is increased from –135 dBm to –120 dBm. Similarly in Figure 7, we have noticed that the CDMA system capacity is unchanged while the Interference Temperature ranges between –135 dBm and –115 dBm. ([1], p. 15).

This passage suggests that Shared Spectrum used an incorrect value for the noise floor, perhaps about –135 dBm. With such a low noise floor, the received signal power values (–120 to –100 dBm) shown represent relatively high power levels, corresponding to greatly reduced coverage, as can be seen by comparing Figure 7 and Figure 8. Shared Spectrum's capacity analysis is therefore unrealistic, and would require a large sacrifice in coverage.

Conclusions
As clearly demonstrated above, Shared Spectrum's claims as conveyed in [1] are based on flawed analysis and are without merit, and indicate a basic lack of understanding about CDMA system design and operation.

References

[1] Appendix A to Reply Comments of Shared Spectrum Company, "The Effect of Interference-Temperature-Based Sharing on CDMA System Capacity," ET Docket 03-237, April 5, 2004.

[2] Jon W. Mark and Weihua Zhuang, *Wireless Communications Networking*, Prentice Hall, Upper Saddle River, NJ, 2003, pp. 223-226.